

1. Exercise 1.8 of the CC Book: Given any graph G on n vertices, we define the following “clique vs. independent set” problem with respect to G : Alice receives as an input C , which is a clique in G (a set of vertices with an edge between any two of them). Bob receives as an input I , which is an independent set in G (a set of vertices with no edges between them). The function $\text{CIS}_G(C, I)$ is defined as the size of $C \cap I$ (observe that this size is either 0 or 1). Prove that for any G , $\Omega(\log n) \leq D(\text{CIS}_G) \leq O(\log^2 n)$. Moreover, if there is a $c < 2$ such that $D(\text{CIS}_G) \leq O(\log^c n)$ for all G , then show that for any $f : X \times Y \rightarrow \{0, 1\}$,

$$D(f) = O\left((\log C^D(f))^c\right).$$

2. Exercise 2.6 of the CC Book: Prove that for all $f : X \times Y \rightarrow \{0, 1\}$ and $z \in \{0, 1\}$, $D(f) \leq C^z(f) + 1$.

3. Exercise 2.23 (1) of the CC Book: Let $\text{Inter} : \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1, \dots, n\}$ be the function that counts the number of entries in which $x_i = y_i = 1$ (think of x and y as subsets of $[n]$, then $\text{Inter}(x, y)$ is the size of their intersection). Show that the rank of M_{Inter} is n . Conclude that for nonboolean functions, the gap between $D(f)$ and $\log \text{rank}(f)$ may be exponential.