

1. Exercise 5.6 of the CC Book (Example 5.5 is helpful): For $\mathbf{x}, \mathbf{y} \in \{0, 1\}^n$, denote by $d(\mathbf{x}, \mathbf{y})$ the Hamming distance between \mathbf{x} and \mathbf{y} (that is, the number of indices in that \mathbf{x} and \mathbf{y} differ). Let R be a relation consisting of all triples $(\mathbf{x}, \mathbf{y}, m)$ such that

$$|m - d(\mathbf{x}, \mathbf{y})| \leq n/3.$$

In other words, computing R is the problem of approximating the Hamming distance between \mathbf{x} and \mathbf{y} . Prove that $D(R) = \Omega(n)$.

2. Exercise 5.11 of the CC Book: Prove that $C^D(R) \geq n^2$ is the best lower bound that can be proven by using Lemma 5.9 (Khrapchenko's bound).

3. Exercise 10.16 of the CC Book (Read Example 10.15 first, where you can find an interesting application of FORK. Also the version here is slightly different from the one in the book since we did not define monotone circuits in class.): Recall the undirected s - t -connectivity function USTCON (Exercise 7.15). The function is similar to STCON but with respect to undirected graphs. That is, given an undirected graph G on n nodes, two of which are marked as s and t , we define $\text{USTCON}(G) = 1$ iff there is a path connecting s and t . We also define M to be the following relation: Alice is given an undirected graph G with $\text{USTCON}(G) = 1$ and Bob is given an undirected graph H with $\text{USTCON}(H) = 0$; find an edge that appears in G but not in H . Prove that $D(M) = \Omega(\log^2 n)$. Hint: Use the relation FORK' (Exercise 5.21).